

■ Kurven sehen und verstehen

Haftendorn April 2017, <http://www.kurven-sehen-und-verstehen.de>

■ Inversion der Steiner-Kurven, Afg 9.15

Inversion am Kreis um $M=(a,b)$, Radius k ,

In[31]= $k = .$

In[32]= $\text{invert} = \left\{ x \rightarrow \frac{k^2 x}{x^2 + y^2}, y \rightarrow \frac{k^2 y}{x^2 + y^2} \right\}$

Out[32]= $\left\{ x \rightarrow \frac{k^2 x}{x^2 + y^2}, y \rightarrow \frac{k^2 y}{x^2 + y^2} \right\}$

In[33]= $\text{invertM} = \left\{ x \rightarrow k^2 \frac{x - a}{(x - a)^2 + (y - b)^2} + a, y \rightarrow k^2 \frac{y - b}{(x - a)^2 + (y - b)^2} + b \right\}$

Out[33]= $\left\{ x \rightarrow a + \frac{k^2 (-a + x)}{(-a + x)^2 + (-b + y)^2}, y \rightarrow b + \frac{k^2 (-b + y)}{(-a + x)^2 + (-b + y)^2} \right\}$

In[51]= $\{a = ., b = ., k = .\};$
 $\text{kurve} = y == x /. \text{invertM}$

Out[52]= $b + \frac{k^2 (-b + y)}{(-a + x)^2 + (-b + y)^2} == a + \frac{k^2 (-a + x)}{(-a + x)^2 + (-b + y)^2}$

In[42]= $\text{FindInstance}\left[b + \frac{k^2 (-b + y)}{(-a + x)^2 + (-b + y)^2} == a + \frac{k^2 (-a + x)}{(-a + x)^2 + (-b + y)^2}, \{a, b, k, x, y\}\right]$
[finde zutreffenden Fall]

Out[42]= $\{\{a \rightarrow 1, b \rightarrow 1, k \rightarrow 0, x \rightarrow 0, y \rightarrow -1\}\}$

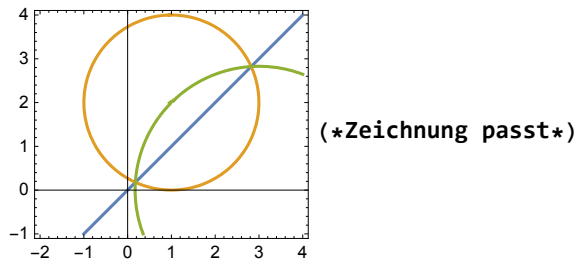
In[44]= $\{a = 1, b = 2, k = 2\};$
 $\text{ContourPlot}\left[\{y == x, (-a + x)^2 + (-b + y)^2 == k^2\},\right.$
[Konturgraphik]

$\left. 2 + \frac{k^2 (-2 + y)}{(-1 + x)^2 + (-2 + y)^2} == 1 + \frac{k^2 (-1 + x)}{(-1 + x)^2 + (-2 + y)^2} \right\}, \{x, -2, 4\}, \{y, -1, 4\},$

$\text{AspectRatio} \rightarrow \text{Automatic}, \text{Axes} \rightarrow \text{True}]$

[Seitenverhältnis [automatisch] [Axen [wahr]

$\{a = ., b = ., k = .\};$



Steinerkurve, invertieren an $M=(a,b)$, Radius k

gemäß Afg 9.15 afg9.15-inv-steiner.ggb

$$\text{In[66]:= steiner} = (9 r^2 + 12 r x + x^2 + y^2)^2 == 4 r (3 r + 2 x)^3$$

$$\text{Out[66]= } (9 r^2 + 12 r x + x^2 + y^2)^2 == 4 r (3 r + 2 x)^3$$

$$\text{In[146]:= invsteiner} = \text{steiner} /. \text{invertM} // \text{FullSimplify}$$

[vereinfache vollst]

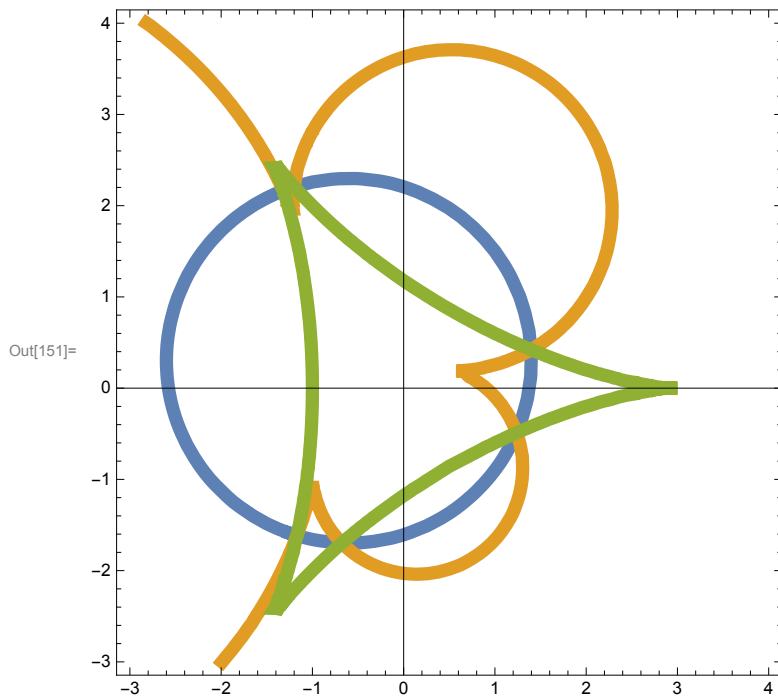
$$\text{Out[146]= } \left(9 + a (12 + a) + b^2 + \frac{k^2 (k^2 - 2 a (6 + a - x) + 12 x + 2 b (-b + y))}{(a - x)^2 + (b - y)^2} \right)^2 ==$$

$$4 \left(3 + 2 \left(a + \frac{k^2 (-a + x)}{(a - x)^2 + (b - y)^2} \right) \right)^3$$

```
In[150]:= {a = -0.6, b = 0.3, r = 1, k = 2};
```

```
ContourPlot[{{(-a + x)2 + (-b + y)2 == k2, invsteiner // Evaluate, steiner // Evaluate},
  Konturgraphik |werte aus |werte aus
  {x, -3, 4}, {y, -3, 4},
  AspectRatio → Automatic, Axes → True, ContourStyle → Thickness[0.02]}
  |automatisch |Axen |wahr |Konturenstil |Dicke
```

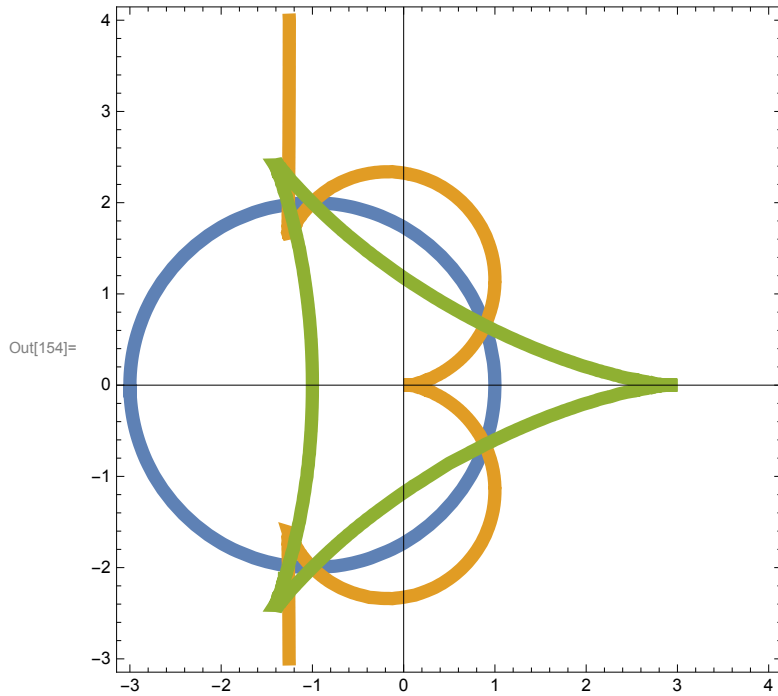
```
{a = ., b = ., k = .};
```



```
In[153]= {a = -1, b = 0, r = 1, k = 2};
```

```
ContourPlot[{{(-a + x)2 + (-b + y)2 == k2, invsteiner // Evaluate, steiner // Evaluate},
  Konturgraphik |werte aus |werte aus
  {x, -3, 4}, {y, -3, 4},
  AspectRatio → Automatic, Axes → True, ContourStyle → Thickness[0.02]}
  |automatisch |Axen |wahr |Konturenstil |Dicke
```

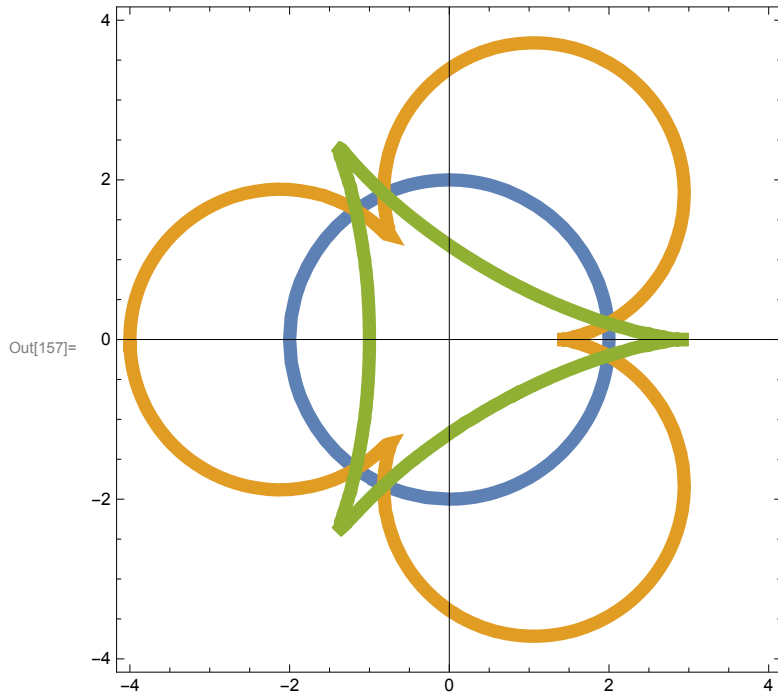
```
{a = ., b = ., k = .};
```



```
In[156]:= {a = 0, b = 0, r = 1, k = 2};
```

```
ContourPlot[{{(-a + x)2 + (-b + y)2 == k2, invsteiner // Evaluate, steiner // Evaluate},
  Konturgraphik |werte aus |werte aus
  {x, -4, 4}, {y, -4, 4},
  AspectRatio → Automatic, Axes → True, ContourStyle → Thickness[0.02]
  |automatisch |Axen |wahr |Konturenstil |Dicke
```

```
{a = ., b = ., k = .};
```



```
In[159]= {a = 1.5, b = 1.3, r = 1, k = 2};
```

```
ContourPlot[{{(-a + x)2 + (-b + y)2 == k2, invsteiner // Evaluate, steiner // Evaluate},
  Konturgraphik |werte aus |werte aus
  {x, -3, 4}, {y, -4, 4},
  AspectRatio → Automatic, Axes → True, ContourStyle → Thickness[0.02]}
  |automatisch |Axen |wahr |Konturenstil |Dicke
```

```
{a = ., b = ., k = .};
```

