Kurven sehen und verstehen

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Kettenlinie und Katenoid, Aufgabe 9.19

Unten steht noch etwas über andere Ansätze

DGL für die Kettenlinie, Schupp hat den Faktor auf der anderen Seite wie Glaeser

$(a\,y'')^2 = 1 + (y')^2$ und das klappt mit y=a csoh($\frac{x}{a}$)

Anpassung für Aufgabe 9.19

In[5]:= `yA = f[2]`

Out[5]= $a \operatorname{Cosh}\left[\frac{2}{a}\right]$

In[13]:= `yA /. a → 0.8`

Out[13]= 4.90583

Hyperbel

In[6]:= $\text{hyp} = y^2 == a^2 \left(1 + \frac{x^2}{b^2}\right)$

Out[6]= $y^2 == a^2 \left(1 + \frac{x^2}{b^2}\right)$

In[17]:= $\text{lb} = \text{Solve}\left[yA^2 == a^2 \left(1 + \frac{2^2}{bq}\right), \{bq\}\right]$

löse

Out[17]= $\left\{\left\{bq \to 4 \operatorname{Csch}\left[\frac{2}{a}\right]^2\right\}\right\}$

In[19]:= $bq = 4 \operatorname{Csch}\left[\frac{2}{a}\right]^2$ `(*KoSekansHyperbolicus=1/ SinusHyperbolicus *)`

Out[19]= $4 \operatorname{Csch}\left[\frac{2}{a}\right]^2$

In[23]:=
```
bq /. a → 0.8
b = √bq /. a → 0.8
```
Out[23]= 0.109275

Out[24]= 0.330567

Parabel

In[45]:= para = y == d x^2 + a

Out[45]= $y == a + d x^2$

In[46]:= $d = \frac{yA - a}{4}$ /. a → 0.8 // N

Out[46]= 1.02646

polynom 4. Grades

In[49]:= poly = y == c x^4 + a;

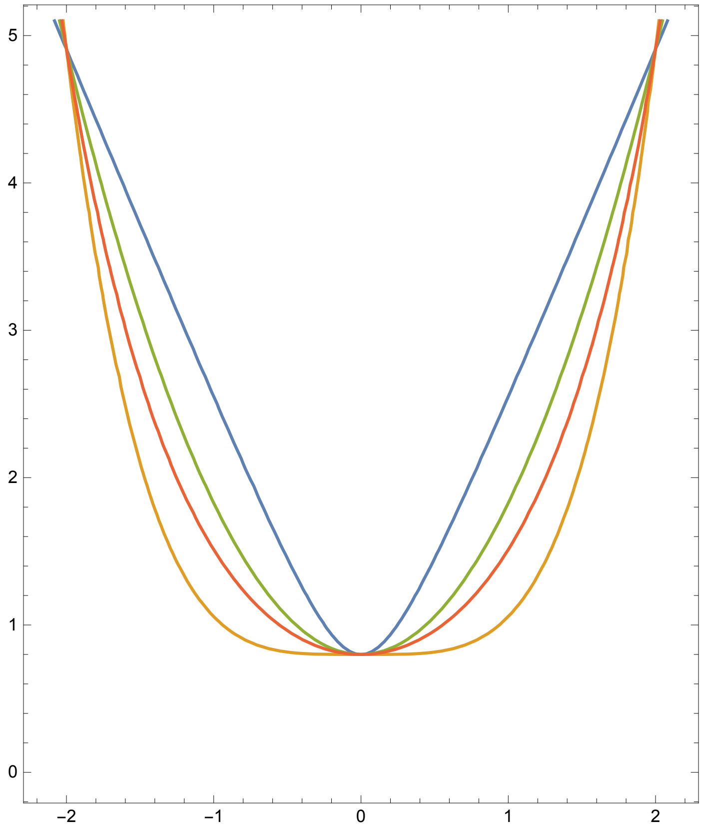
In[27]:= $c = \frac{yA - a}{16}$ /. a → 0.8 // N

Out[27]= 0.256614

alle

In[54]:=
```
a = 0.8;
ContourPlot[{hyp // Evaluate, poly // Evaluate, para // Evaluate, y == f[x]},
 {x, -2.2, 2.2}, {y, -0.1, 5.1}, AspectRatio → Automatic]
a =.
```
Konturgraphik, werte aus, werte aus, werte aus, Seitenverhältnis, automatisch

Out[54]=



■ Volumina

Volumen Katenoid

In[56]:=
```
2 Pi Integrate[(a Cosh[x / a]) ^2, x]
```
integriere, Kosinus Hyperbolicus

Out[56]= $2\, a^2 \pi \left(\frac{x}{2} + \frac{1}{4} a \operatorname{Sinh}\left[\frac{2x}{a}\right]\right)$

$2\, a^2 \pi \left(\frac{x}{2} + \frac{1}{4} a \operatorname{Sinh}\left[\frac{2x}{a}\right]\right)$

Sinus Hyperbol

In[57]:=
```
2 Pi Integrate[(a Cosh[x / a]) ^2, {x, 0, 2}] /. a → 0.8
```
integriere, Kosinus Hyperbolicus

Out[57]= 63.699

Volumen Hyperboloid

In[62]:=
```
2 Pi Integrate[a^2 (1 + x^2 / b^2), {x, 0, 2}] /. a → 0.8
```

Out[62]= 106.174

Volumen Paraboloid

In[63]:=
```
2 Pi Integrate[(d x^2 + a)^2, {x, 0, 2}] /. a → 0.8
```

Out[63]= 77.9284

Volumen Quartik

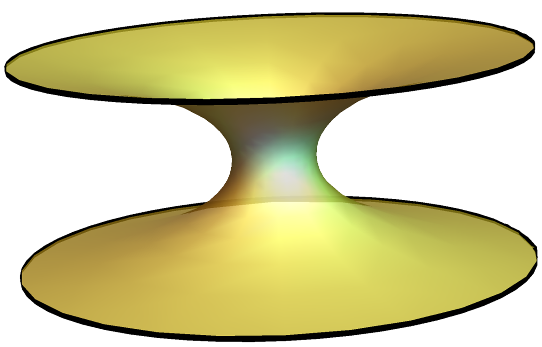
In[66]:=
```
2 Pi Integrate[(c x^4 + a)^2, {x, 0, 2}] /. a → 0.8
```

Out[66]= 48.091

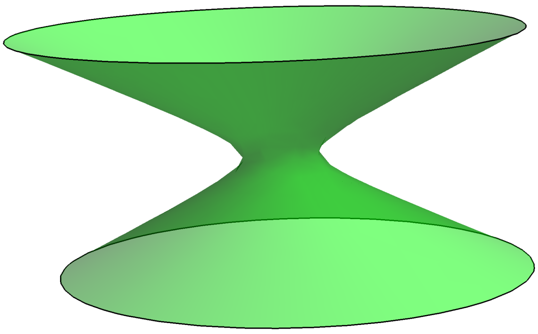
3D-Ansichten

3D Katenoid

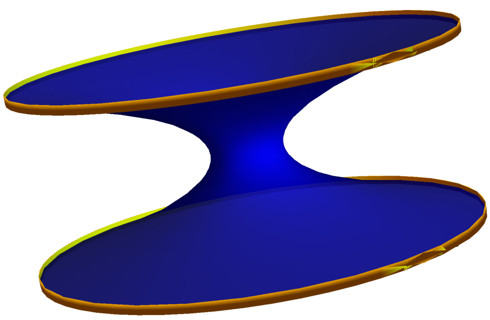
```
katenoid = ParametricPlot3D[{{ye Cos[u], ye Sin[u], 2},
  {ye Cos[u], ye Sin[u], -2}, {a Cosh[v / a] Cos[u], a Cosh[v / a] Sin[u], v}},
  {v, -2, 2}, {u, 0, 2 Pi}, Mesh → None, Boxed → False, Axes → False,
  PlotStyle → {{Thickness[0.1], Black}, {Thickness[0.1], Black},
    {Opacity[0.8], ColorFunction → Function[{x, y, z}, Hue[z]]}}]
```



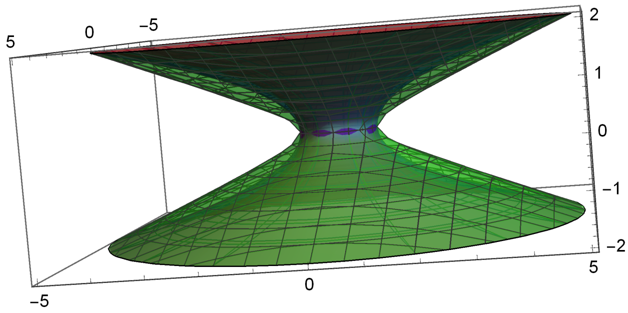
```
hyperboloid =
 ContourPlot3D[+x^2 / a^2 + y^2 / a^2 - z^2 / b^2 == 1, {x, -5, 5}, {y, -5, 5}, {z, -2, 2},
 |3D-Konturgraphik
  Mesh → None, Boxed → False, Axes → False, ContourStyle → {Opacity[0.5],  Green
  |Gitter… |keine |einger… |falsch |Axen |falsch |Konturenstil  |Deckkraft  |grün
   }, AspectRatio → 0.5]
    |Seitenverhältnis
```



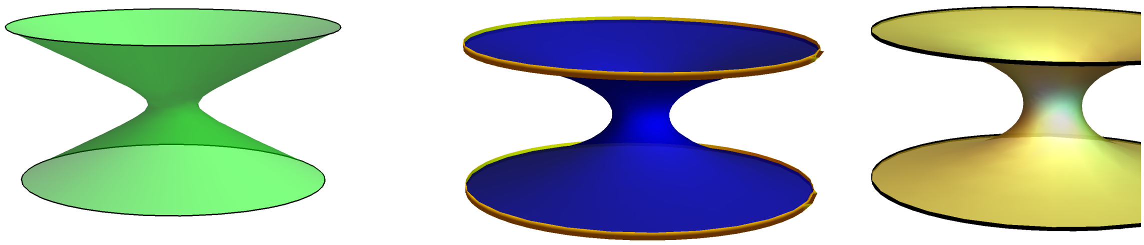
```
paraboloid = ParametricPlot3D[{{ye Cos[u], ye Sin[u], 2}, {ye Cos[u], ye Sin[u], -2},
             |parametrische 3D-Darstellung |Kosinus |Sinus  |Kosinus |Sinus
   {(1 / (2 p) v^2 + a) Cos[u], (1 / (2 p) v^2 + a) Sin[u], v}}, {v, -2, 2}, {u, 0, 2 Pi},
                        |Kosinus                  |Sinus                          |Kreisza
  Boxed → False, Axes → False, Mesh → None, PlotStyle → {{Thickness[0.2], Yellow},
         |falsch   |Axen  |falsch |Gitter… |keine |Darstellungsstil |Dicke  |gelb
    {Thickness[0.2], Yellow}, {Opacity[0.9], Lighter, Blue }}]
    |Dicke            |gelb     |Deckkraft      |heller   |blau
```



```
Show[paraboloid, hyperboloid, katenoid]
```
zeige an




```
GraphicsRow[{ hyperboloid, paraboloid, katenoid}]
```
Graphik in Zeilenanordnung



```
Export[
  "C:\\Users\\User\\Kurven-erkunden-und-verstehen\\Mathematica-kurven\\katenoid+.eps",
  %88, "EPS"]
```
exportiere

Konstante

C:\Users\User\Kurven-erkunden-und-verstehen\Mathematica-kurven\katenoid+.eps

Mantelflächen M= 2 π Integrate$\left[f(x)\sqrt{1+f'(x)^2}, \{x, a, b\}\right]$

In[67]:=
```
M[x_] := 4 π Integrate[g[x] √(1 + g'[x]), {x, 0, 2}]
```
integriere

Mantel Katenoid

```
a =.
```

In[70]:= $f[x]\sqrt{1+f'[x]^2}$ // Simplify // PowerExpand
vereinfache   multipliziere Potenzen aus

Out[70]= $a\,\mathrm{Cosh}\left[\frac{x}{a}\right]^2$

4π Integrate$\left[a\left(\mathrm{Cosh}\left[\frac{x}{a}\right]\right)^2, x\right]$
integriere

Out[71]= $4\,a\,\pi\left(\frac{x}{2}+\frac{1}{4}\,a\,\mathrm{Sinh}\left[\frac{2\,x}{a}\right]\right)$

Mka = 4π Integrate$\left[a\left(\mathrm{Cosh}\left[\frac{x}{a}\right]\right)^2, \{x, 0, 2\}\right]$
integriere

Out[72]= $a\,\pi\left(4+a\,\mathrm{Sinh}\left[\frac{4}{a}\right]\right)$

In[73]:= Mk = Mka /. a → 0.8

Out[73]= 159.248

Mantel Paraboloid

In[87]:= d =.

In[88]:= par[x_] := d x^2 + a; par[x]

Out[88]= $a+d\,x^2$

In[83]:= $\mathrm{par}[x]\sqrt{1+\mathrm{par}'[x]^2}$ // Simplify // PowerExpand
vereinfache   multipliziere Potenzen aus

Out[83]= $\left(a+d\,x^2\right)\sqrt{1+4\,d^2\,x^2}$

In[84]:= 4π Integrate$\left[\mathrm{par}[x]\sqrt{1+\mathrm{par}'[x]^2}, x\right]$ // Simplify
integriere   vereinfache

Out[84]= $\frac{1}{16\,d^2}\pi\left(2\,d\,x\sqrt{1+4\,d^2\,x^2}\left(1+16\,a\,d+8\,d^2\,x^2\right)+\left(-1+16\,a\,d\right)\mathrm{ArcSinh}[2\,d\,x]\right)$

In[90]:= $d=\frac{yA-a}{4}$ /. a → 0.8 // N
nı

Out[90]= 1.02646

In[130]:= Mpa = 4π Integrate$\left[\mathrm{par}[x]\sqrt{1+\mathrm{par}'[x]^2}, \{x, 0, 2\}\right]$ /. a → 0.8 // Simplify
integriere   vereinfache

Out[130]= 159.53

Mantel Hyperboloid

In[95]:= $hy[x\_] := a\sqrt{1+\frac{x^2}{bb\hat{}2}}$

In[96]:= $hy[x]\sqrt{1+hy'[x]^2}$ // Simplify // PowerExpand
vereinfache multipliziere Potenzen aus

Out[96]= $a\sqrt{1+\frac{x^2}{bb^2}}\sqrt{1+\frac{a^2x^2}{bb^4+bb^2x^2}}$

In[98]:= 4π Integrate$\left[hy[x]\sqrt{1+hy'[x]^2}, x\right]$ // FullSimplify
integriere vereinfache vollstä

Out[98]= $2a\pi\left(x\sqrt{1+\frac{x^2}{bb^2}}\sqrt{1+\frac{a^2x^2}{bb^4+bb^2x^2}}+\frac{1}{\sqrt{a^2+bb^2}}\right.$
$\left.bb^2\,\text{Log}\left[a^2x+bb^2\left(x+\sqrt{a^2+bb^2}\sqrt{1+\frac{x^2}{bb^2}}\sqrt{1+\frac{a^2x^2}{bb^4+bb^2x^2}}\right)\right]\right)$

In[99]:= bq /. a → 0.8
b = $\sqrt{bq}$ /. a → 0.8

Out[99]= 0.109275

Out[100]= 0.330567

In[129]:= Mhy = 4π Integrate$\left[hy[x]\sqrt{1+hy'[x]^2}\right.$ /. {a → 0.8, bb → b}, {x, 0, 2}]
integriere

Out[129]= 161.779

Mantel Quartik

In[105]:= c =.

In[106]:= pol[x_] := c x^4 + a; pol[x]

Out[106]= $a+c\,x^4$

In[108]:= $pol[x]\sqrt{1+pol'[x]^2}$ // Simplify // PowerExpand
vereinfache multipliziere Potenzen aus

Out[108]= $\left(a+c\,x^4\right)\sqrt{1+16\,c^2x^6}$

In[122]:= c = $\frac{yA-a}{16}$ /. a → 0.8 // N
nu

Out[122]= 0.256614

```
a = 0.8; 4 π Integrate[pol[x] √(1 + pol'[x]^2), x] // Simplify
                  integriere                          vereinfache

Out[123]= 0.403089 (6.23503 x + 1. x^5) √(1. + 1.05362 x^6) +
  7.53982 x Hypergeometric2F1[0.166667, 0.5, 1.16667, -1.05362 x^6] +
  0.241853 x^5 Hypergeometric2F1[0.5, 0.833333, 1.83333, -1.05362 x^6]

In[126]:= Mpol = 2 π Integrate[pol[x] √(1 + pol'[x]^2), {x, -2, 2}] // Chop
                    integriere                                        ersetz

Out[126]= 159.715 - 4.17265 × 10^-8 ⅈ

In[132]:= {Mhy, Mpa, Mpol, Mk}

Out[132]= {161.779, 159.53, 159.715 - 4.17265 × 10^-8 ⅈ, 159.248}

In[131]:= {(Mhy - Mk)/Mk, (Mpa - Mk)/Mk, (Mpol - Mk)/Mk}

Out[131]= {0.0158982, 0.00177134, 0.00293278 - 2.62023 × 10^-10 ⅈ}
```

Andere Ansätze

Versuch mit b